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# Exam FAM-L Study Manual

1st Edition, 3rd Printing

Abraham Weishaus, Ph.D., FSA, CFA, MAAA

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1st Edition, 3rd Printing

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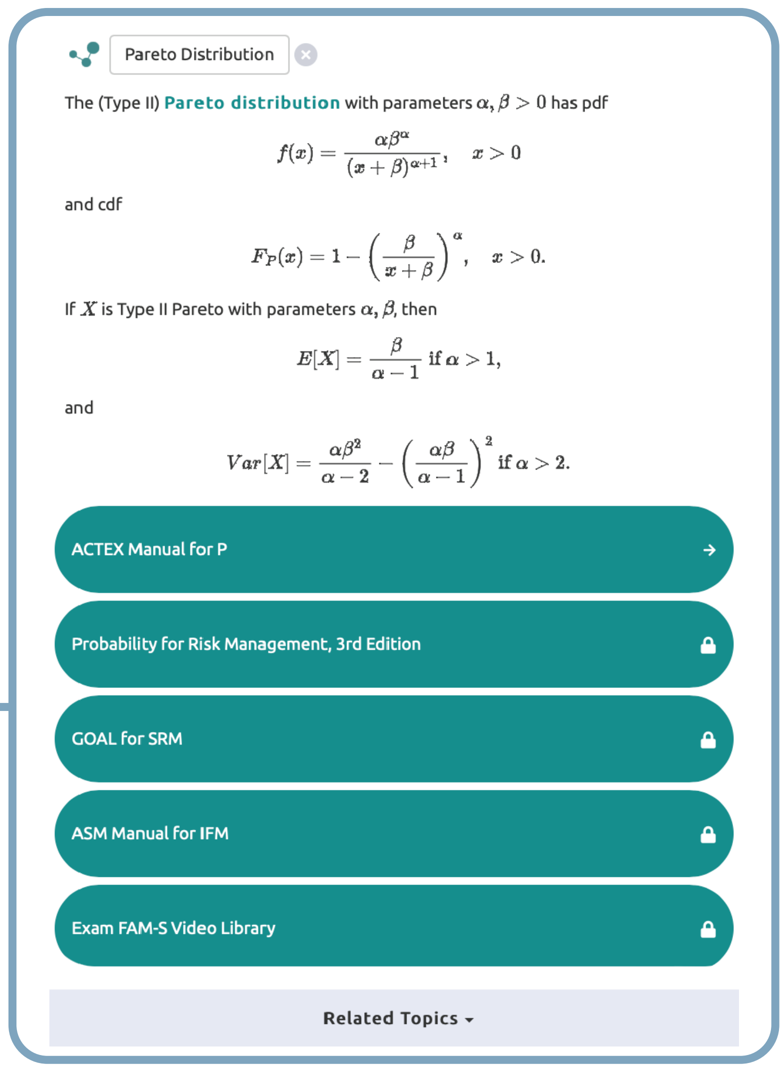

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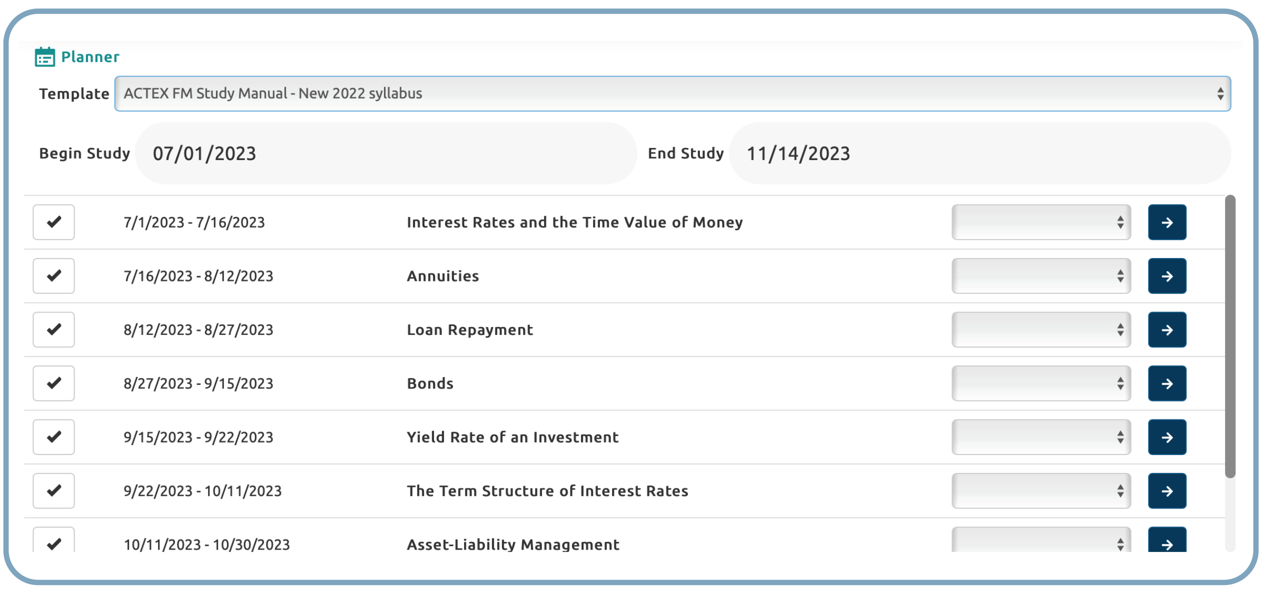


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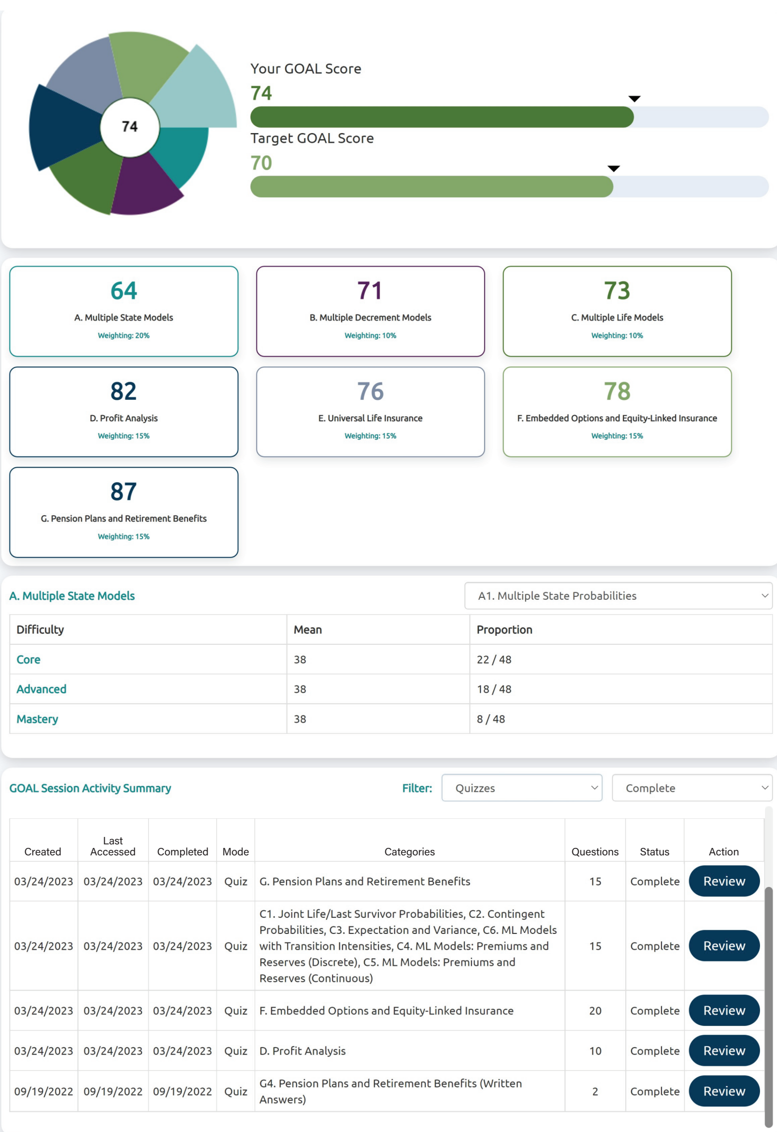


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| Created | Last Accessed | Completed | Mode | Categories | Questions | Status | Action |
|---|---|---|---|---|---|---|---|
| 03/24/2023 | 03/24/2023 | 03/24/2023 | Quiz | G. Pension Plans and Retirement Benefits | 15 | Complete | Review |
| 03/24/2023 | 03/24/2023 | 03/24/2023 | Quiz | C1. Joint Life/Last Survivor Probabilities, C2. Contingent Probabilities, C3. Expectation and Variance, C6. ML Models with Transition Intensities, C4. ML Models: Premiums and Reserves (Discrete), C5. ML Models: Premiums and Reserves (Continuous) | 15 | Complete | Review |
| 03/24/2023 | 03/24/2023 | 03/24/2023 | Quiz | F. Embedded Options and Equity-Linked Insurance | 20 | Complete | Review |
| 03/24/2023 | 03/24/2023 | 03/24/2023 | Quiz | D. Profit Analysis | 10 | Complete | Review |
| 09/19/2022 | 09/19/2022 | 09/19/2022 | Quiz | G4. Pension Plans and Retirement Benefits (Written Answers) | 2 | Complete | Review |

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# Contents

## Appendices 921

# Preface

Welcome to Exam FAM-L!

## Syllabus

The Fall 2022 syllabus is posted at the following URL:

https://www.soa.org/497029/globalassets/assets/files/edu/2022/2022-10-exam-fam-syllabus.pdf

The topics are

1. Survival models
2. Insurances
3. Annuities
4. Premiums
5. Reserves
6. Estimating mortality rates

The textbook for the course is *Actuarial Mathematics for Life Contingent Risks* third edition. This is a college-style textbook. It is oriented towards practical application rather than exam preparation. Almost all exercises require use of spreadsheets or derivation of formulas.

The syllabus splits the material into five broad topics and states percentage ranges for the topics. In the following table, I've doubled the percentages to account for FAM-L being half an exam.

| Topic | Weight | Lessons in Manual |
|---|---|---|
| Insurance Coverages and Retirement Financial Security Programs | 5–15% | 2 |
| Mortality Models | 15–25% | 3–9 |
| Parametric and Non-Parametric Estimation | 10–20% | 10–13 |
| Present Value Random Variables for Long-Term Insurance Coverages | 20–30% | 14–25 |
| Premium and Policy Value Calculation for Long-Term Insurance Coverages | 25–35% | 26–41 |

For a 20-question exam, each 5% represents one question.

## Other downloads from the SOA site

### Tables

*Download the tables you will be given on the exam.* They will often be needed for the exercises. They are currently posted at

https://www.soa.org/4a1b80/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-tables.pdf

The tables include the Standard Ultimate Life Table with insurance functions, and some interest functions. The tables do not include a standard normal distribution table. Instead, you will use a Prometric calculator at the exam, which will provide standard normal distribution and the inverse of that function to 5 decimal places.

Another set of tables, the tables from the former MLC, will be useful if you wish to work on pre-2018 exam questions that use the Illustrative Life Table. You can find it at https://www.soa.org/Files/Edu/edu-2013-mlc-tables.pdf

However, I have converted all pre-2012 exam questions to use the Standard Ultimate Life Table, and the SOA converted questions from 2012 and later when they incorporated them in their sample questions. So it is unlikely you'll need the Illustrative Life Table.

### Notation and terminology note

The notation and terminology note is at

https://www.soa.org/4a2744/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-notation.pdf

In almost all cases, the exam uses the terminology of *Actuarial Mathematics for Life Contingent Risks*. This manual uses the terminology that will be used on the exam.

The textbook uses the unusual name "policy value" for "reserve". On LTAM and earlier exams, the exams used the term "reserve", but the notation and terminology note states that "policy value" will be used on FAM-L for the expected value of future loss. "Reserve" will only be used for the capital a company puts aside to cover future losses.

### Sample questions

Sample questions are at

https://www.soa.org/4a3519/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-quest.pdf

and their solutions are at

https://www.soa.org/4a3522/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-sol.pdf

At the end of each lesson in this manual, you will find a list of multiple-choice sample questions related to the material of the lesson if there are any. The questions and solutions themselves are not included in this manual. However, solutions to the original exam questions (which, for MLC, used the Illustrative Life Table) are in Appendix B.

## Old exam questions in this manual

There are about 380 original exercises in the manual and about 700 old exam questions. The old exam questions come from old Part 4, Part 4A, Course 150, Course 151 exams, 2000-syllabus Exam 3, Exam C, Exam M, and Exam MLC. However, very few questions from the 2012 and later MLC exams are given in the exercises, so you may use those exams or the SOA sample questions as final practice.

SOA Part 4 in 1986 had morning and afternoon sessions. I indicate afternoon session questions with "A". The morning session had the more basic topics (through reserves), while the afternoon session had advanced topics (multiple lives, multiple decrements, etc.) Both sessions were multiple choice questions.

SOA Course 150 from 1987 through 1991 had multiple choice questions in the morning and written answer questions in the afternoon. Since LTAM will include written answer questions, I've included all applicable written answer questions in the exercises.

The CAS Part 4A exams awarded varying numbers of points to questions; some are 1 point and some are 2 points. The 1 point questions are probably too easy for a modern exam, but they'll give you a little practice. The pre-1987 exams probably were still based on Jordan (the old textbook), but the questions I provided, while ancient, still have value. Similarly, the cluster questions on SOA Course 150 in the 1990s generally were awarded 1 point per question.

Although the CAS questions are limited to certain topics, are different stylistically, and are easier, they are a good starting point.

Course 151 is the least relevant to this subject. I've only included a small number of questions from 151 in the first lesson, which is background.

Back in 1999, the CAS and SOA created a sample exam for the then-new 2000 syllabus. This exam had some questions from previous exams but also some new questions, some of them not multiple choice. This sample exam

**Table 1:** 9 Week Study Schedule for Exam LTAM

| Subject | Lessons | Study Period | Hard/Long Lessons | Easy/Short Lessons |
|---|---|---|---|---|
| Types of Long Term Products | 2 | 0.5 weeks | | |
| Survival Distributions | 3–9 | 2 weeks | 4,8 | 3 |
| Estimation | 10–13 | 1 week | | 10 |
| Insurances | 14–19 | 1 week | 14,18 | 19 |
| Annuities | 20–25 | 1 week | 20,23 | 25 |
| Premiums | 26–34 | 1.5 weeks | 28,33 | |
| Reserves, Part I | 35–37 | 1 week | | |
| Reserves, Part II | 38–41 | 1 week | 39, 41 | |

was never a real exam, and some of its questions were defective. This sample exam is no longer available on the web. I have included appropriate questions from it. *Whenever an exercise is labeled 1999 C3 Sample, it refers to the 1999 sample, not the current list of sample questions.*

Questions from CAS exams given in 2005 and later are not included in this manual. There is a lot of better practice material available, so in order to make this manual a little less bulky, I do not provide solutions to old CAS 3, 3L, and LC exams.

Questions from old exams are marked xxx:yy, where xxx is the time the exam was given, with S for spring and F for fall followed by a 2-digit year, and yy is the question number. Sometimes xxx is preceded with SOA or CAS to indicate the sponsoring organization. From about 1986 to 2000, SOA exams had 3-digit numbers (like 150) and CAS exams were a number and a letter (like 4A). From 2000 to Spring 2003, the exams were jointly sponsored. There was a period in the 1990s when the SOA, while it allowed use of its old exam questions, did not want people to reveal which exam they came from. As a result, I sometimes had study notes for old exams in this period and could not identify the exam they came from. In such a case, I mark the question aaa-bb-cc:yy, where aaa-bb-cc is the study note number and yy is the question number. Generally aaa is the exam number (like 150), and cc is the 2-digit year the study note was published.

## Characteristics of this exam

The exam will have 20 multiple choice questions worth 2 points apiece You will be given 1.75 hours to complete the exam, or 5.25 minutes per question.

Ttere is no penalty for guessing. Fill in all questions regardless of whether you have time to work out the question or not—you lose nothing and you may be lucky!

The answer choices on SOA exams are almost always specific answers, not ranges.

## Study schedule

Although this manual seems huge, much of it is exercises and practice exams. You do not have to do every exercise; do enough to gain confidence with the material. With intense studying, you should be able to cover all the material in 4 months.

It is up to you to set up a study schedule. Different students will have different speeds and different constraints, so it's hard to create a study schedule useful for everybody. However, I offer a sample 9-week study schedule, Table 1, as a guide. The amount of time you spend on this lesson depends on the strength of your probability background. You may decide to skip it and refer to it as needed.

The study schedule lists lessons that are either long or hard, as well as those that are short or easy or just background, so that you may better allocate your study time within the study periods provided for each subject.

## Acknowledgements

I would like to thank the SOA and CAS for allowing me to use their old exam questions. I'd also like to thank Harold Cherry for suggesting this manual and for providing three of the pre-2000 SOA exams and all of the pre-2000 CAS exams I used.

The creators of TEX, LATEX, and its multitude of packages all deserve thanks for making possible the professional typesetting of this mathematical material.

## Errata

Please report all errors you find in these notes to the author. You may send them to the publisher at mail@studymanuals.com or directly to me at errata@aceyourexams.net. Please identify the manual and edition the error is in. This is the 1st edition of the Exam FAM-L manual.

An errata list will be posted at errata.aceyourexams.net. Check this errata list frequently.

## Flashcards

Many students find flashcards a useful tool for learning key formulas and concepts. ASM flashcards, available from the same distributors that sell this manual, contain the formulas and concepts from this manual in a convenient deck of cards. The cards have cross references, usually by page, to the manual.

# Lesson 2

# Introduction to Long Term Insurance

**Reading:** *Actuarial Mathematics for Life Contingent Risks* 3rd edition

A long-term insurance contract is a contract in which the insurer makes commitments that extend for more than one year. Generally the insurer offers insurance for a long period of time at a guaranteed premiums, or at least limits the degree to which the premium can be changed.

Life insurance is the long-term insurance contract we will concentrate on for most of this course. It is the simplest long-term insurance since the only contingency to consider is death. Any long-term contract has to take death into account; at the very least, death terminates the contract. If other contingencies are involved (like disability or illness), the contract will be more complicated than a life insurance contract.

The first chapter of the textbook is an introduction to long-term insurance. It discusses the features of these insurances, but not the mathematics. The following is a summary of the chapter. While I've tried to include anything important, you may want to read the chapter in the textbook to assure yourself that you have learned every little piece of trivia that could be asked on the exam.[1] [2]

Life insurance pays a lump sum, known as a *sum insured*, upon death. (In the U.S., this amount may also be called the face amount of the policy.) The insured pays a premium at the beginning of the period, often the year, for this insurance. Originally, a group of individuals of a specific age or in a specific age group paid an amount so that the total amount collected equalled the total benefits paid. This is called ***assessmentism***. Group life insurance still uses assessmentism. But for individuals, this method means that premiums increase with age. People are likely to withdraw from the group when the premium gets too high.

This led to the level premium insurance contract. In this contract, the same premium is paid each year. These contracts are more complex since the higher premiums collected in the earlier years must be invested to pay for the higher benefits of the later years.

In order to purchase life insurance, one must have ***insurable interest*** in the life insured. One has to be related to the insured life, or have a close business relationship with that person. Otherwise the insurance is a wager on someone's death, which is not desirable.

**Traditional insurance contracts** Traditional insurance contracts have fixed premiums and fixed benefits. Life insurance provides a benefit upon death; annuities provide a periodic payment while the annuitant is alive. For life insurance products, premiums are payable periodically (annually, semi-annually, quarterly, or monthly) throughout the life of the contract. However, the premium payment period often is limited to a certain number of years or until the insured reaches a certain age, because older insureds may be less able to pay the premium. For annuities, a single premium may be paid. Otherwise, periodic premiums are payable during a deferral period; the annuity benefits begin after the deferral period. You would not typically have money going both ways at the same time—premiums payable while the annuitant is receiving benefits.

Examples of traditional insurance contracts are

**Term insurance** Term insurance pays a benefit if the policyholder dies within a fixed amount of time. It is useful for insuring a family at a low cost, or insuring a business against losses due to death of an employee. In the latter case, the company pays the premiums and receives the benefits. This type of insurance may be called *key person insurance* or *COLI*–company owned life insurance.

[1]Examples of trivia I've not included are:

1. In the UK, when is it called insurance and when is it called assurance?
2. Which insurance rider inspired a Hollywood feature?

[2]in this lesson, footnotes are my own comments on the textbook, and occasionally modify the textbook. They are purely to give you the situation in the U.S.; the textbook tends to reflect the international situation. On exams do not use anything from these footnotes!

Other types of term insurance are

1. Decreasing term insurance to pay off the balance of a mortgage.
2. **Renewable term insurance** allows renewal at the expiry date, at an increased premium.
3. **Convertible term insurance** allows conversion of the policy to whole life.

**Whole life** **Whole life insurance** pays a benefit whenever the policyholder dies. Premiums are level. The policy has a cash value that is paid if the policyholder surrenders the policy.

In the U.S., non-forfeiture laws require cash surrender values to be offered on whole life insurance. These cash values increase over the years of the policy, but are less than the face amount.[3]

Some uses of whole life insurance are:

1. Payment of funeral expenses.
2. Payment of estate taxes.
3. Saving vehicle for younger lives.

**Endowment insurance** **Endowment insurance** pays a benefit if the policyholder dies within a certain period. If the policyholder survives to the end of the period, the benefit is paid at that point. It has cash values. Endowment insurance is not offered in North America or the U.K. because the investment component does not compete with other investments; the returns are low and the investment is not flexible.[4] But endowment insurance is popular in developing nations.

**Joint life/multiple life insurance** This type of insurance pays upon the first death or the last death in a group of individuals. **Joint life** is a policy on husband and wife; companies may insure their officers using multiple life insurance. According to the textbook first-to-die policies are more common than last-to-die policies.[5]

**Participating insurance** Whole life insurance must make interest assumptions into the distant future. In order not to lose money, these assumptions are conservatively low. But the product is then unattractive. To compensate, **participating insurance** (par insurance for short) pays non-guaranteed dividends based on the excess of the rate earned on investments over the assumption. These dividends are known as bonuses outside North America.

In North America, policyholders may choose to receive these dividends in many forms, such as

**Cash refunds** Dividend is paid as cash.

**Premium reductions** Dividend is offset against premium. This is really no different from a cash refund.

**Increased paid up insurance** The death benefit is increased.[6]

In the U.K., dividends are used only to increase the death benefit, the third option above. The dividends are called ***reversionary bonuses***, and come in three variations[7]:

**simple reversionary bonuses** Applied to original face amount only

**compound reversionary bonuses** Applied to sum of original face amount and prior bonuses

**super-compound reversionary bonuses** Two different bonus rates, one for original face amount, another for prior bonuses.

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[3]If they were greater than the face amount, a policyholder on his death bed would surrender the policy.

[4]And importantly, the tax laws offer very unfavorable treatment to these policies (but the book doesn't mention this).

[5]But in the U.S., last-to-die products on husband/wife are especially popular as a method of paying estate taxes, due to their low premiums. Generally estates pass tax-free to a spouse, so the benefit is not needed until the second death.

[6]Other options not mentioned in the textbook are (1) Leave the dividend with the company to accumulate with interest, (2) purchase one-year term insurance; in other words, increase the death benefit for one year only

[7]In the U.S., dividends used to purchase paid up additions follow the "super-compound reversionary bonus" style.

Terminal bonuses[8] are paid on policies that terminate due to death or surrender.

Advantages/disadvantages of cash dividends versus reversionary bonuses are

1. Cash dividends allow policyholders to use them to pay premiums, which may be helpful if policyholder is short of cash.
2. Cash dividends may be taxable.
3. Cash dividends are simpler to understand.
4. Reversionary bonuses are more consistent with the purpose of the policy.
5. Companies may only give some portion of reversionary bonuses to policyholders who surrender, which is unfair to them.[9]
6. Reversionary bonuses make it easier for companies to smooth their dividends.
7. Cash dividends are expensive to operate.
8. Cash dividends require liquidity for the insurer, restricting its investments somewhat.

**Additional benefits that policies may provide**

**Policy loans** For policies with cash values, a **policyholder may take a loan against the cash value**. This loan would be deducted against the benefit if it is not repaid before death.

**Accelerated death benefits** Under this rider, the **death benefit is paid upon terminal illness**.

**Accidental death benefit** Under this rider, also known as *double indemnity*, **twice the death benefit is paid if death is due to accident**.

**Disability waiver** Under this rider, the **premium is waived upon disability**.

**Family income** This rider is term insurance. Under this rider, upon death, **a fixed payment is made each month until the expiry of the rider**. For example, if the rider is for 10 years and the insured dies within 8 years, the policy pays a benefit for 2 years. Thus this is decreasing term insurance. According to the textbook, the base policy is a term insurance for the same number of years.

**Reasons the design of life insurance contracts has changed recently**

1. Competition with mutual funds and banks for policyholders' savings. Thus insurance products have investment components.
2. Changing demographics and lifecycles.
3. Developments in financial science and technology. Modern financial science improves risk management techniques. Computer technology allow more complex products.
4. Better informed customers. Smarter customers require more competitive products.

**Nontraditional insurance contracts** Here are examples of non-traditional products. We do not discuss non-traditional products any further in this course

1. **Universal Life**. This product has flexible premiums. The benefit payable upon death may also be adjustable.

   Premiums go into a notional account. It is notional only; there are no designated assets. The insurer subtracts mortality and expense charges and credits interest. The credited interest rate may change from time to time, but cannot be less than 0%. Generally charges and credits are made monthly. The policy stays in force as long as the account balance is positive. In the earlier years, there is a surrender charge if a policyholder surrenders the policy.

[8]In North America they're called terminal dividends.

[9]In the U.S., this is not allowed. Once a policyholder gets a paid-up addition, it cannot be taken back.

2. **Unitized-with-profit.** This is a UK product. In the UK, products that allow policyholders to share in the company's profits (which are called "participating" in the US) are called "with-profit". In unitized-with-profit contracts, the premium purchases units of investments, which earn reversionary bonuses, but cannot decrease in value.

3. Equity-linked insurance. Two types[10]:

   (a) **Variable annuity** (called "unit-linked" in the UK) invests premiums in mutual funds. Credited amounts may be positive or negative, depending on investment performance.

   (b) **Equity-indexed annuity** has guaranteed returns, and the policyholder receives a portion of the growth on an index (e.g. a stock index) if greater than the guarantee.

**Distribution methods** Most insurance is sold through brokers. Brokers are paid commissions, which are a percentage of premium. Usually the first year commission is a higher percentage of premium than renewal commissions. This is referred to as a *front end load*.

*Direct marketing* is an alternative to brokers. Typically policies sold by direct marketing are for small face amounts and are not as heavily underwritten as broker-sold policies. Hence mortality is higher, but expenses are lower both due to simplified underwriting and lack of commissions. Often such policies are for funeral expenses or credit insurance—insurance paying off a loan if the insured dies.

**Underwriting classes** When a company sells insurance, it usually **underwrites** the policyholder; it determines how healthy the policyholder is and sets premiums based on the policyholder's underwriting class. Typically the underwriting classes are:

1. Preferred: low risk factors, no smoking or alcohol abuse, no high-risk occupations or hobbies. Preferred classes are common in North America, but not in the rest of the world.

2. Normal[11]: some risk factors, but insurable at standard rates.

3. Rated: Elevated risk factors, higher than standard rates must be charged.

4. Uninsurable: Too risky, not insurable at any price.

For life insurance, there is adverse selection by policyholders; the sicker one is, the more likely one is to buy life insurance, and for higher amounts. Underwriting therefore is stricter the higher the sum insured is. Underwriting may only require a policyholder statement for a small amount, but will require a physician's statement for a larger amount and may require examination by the company's doctor for very large amounts.

**Types of annuities** An annuity makes a periodic payment to the purchaser of the annuity. Types of annuities are:

1. Single Premium Deferred Annuity (SPDA). Policyholder pays a single premium. Annuity commences payments on some future date.

2. Single Premium Immediate Annuity (SPIA). Policyholder pays a single premium and annuity payments commence immediately. For example, if payments are monthly, the first payment is one month after the premium is paid.

3. Regular Premium Deferred Annuity (RPDA). Policyholder pays premium on a regular basis until annuity payments commence.

These three types categorize annuities based on when the premiums and benefits are paid. The next three types categorize when benefits are paid for annuities on two lives.

4. Joint life annuity. Annuity makes payments only when both are alive.

[10]For unknown reasons, the textbook discusses annuities here rather than insurances.
[11]Called "standard" in the U.S.

5. Last survivor annuity. Annuity makes payments when at least one is alive.

6. Reversionary annuity. Annuity makes payments to second life after first life dies.

A *guaranteed annuity* is one that makes payments for a period regardless of whether the annuitant is alive. Another name for this annuity is a "certain and life annuity".

Life annuities during their payment period do not have a surrender value. Allowing surrender of such annuities would lead to adverse selection, since individuals who believe they will die soon will surrender their annuities. The pricing of annuities assumes that some people will die earlier, leaving funds to pay annuitants who live longer.

Annuities are not underwritten. Insurance companies make more money if the annuitant dies earlier. It is generally not possible to collect evidence that an annuity purchaser will not live too long.[12]

**Disability income insurance** **Disability insurance** pays a monthly benefit while the policyholder is disabled. Benefit is a proportion of salary, generally 50–70%, to encourage return to work. Insurance may continue until retirement.

Some time provisions of disability insurance are:

**Waiting period** , or *elimination period*. Benefits only begin after insured is disabled for this period. Typically 30, 60, 180, or 365 days.

**Benefit period** . Benefits are payable for at most a certain amount of time. Time is measured from when benefits start, after the waiting period. Typical options are 2 years, 5 years, or to age 65.

**Off period** . This is the amount of time that must elapse between two disability periods before they are considered separate. This may work both for and against the policyholder; if the off period has not passed, there is no new waiting period, but there is no new benefit period either; the second period's benefits would only be for the remainder of the benefit period minus the first disability's period.

Some other characteristics of disability insurance are:

- If the policyholder can do some work but not full-time, the policyholder may be eligible for a lower benefit for partial disability.
- Benefit may be reduced if policyholder receives benefits from other sources, like the government.
- Own job versus any job. Own job insurance pays benefits if the disabled person can't return to his job, even if other work is available. Any job insurance only pays if the disabled person is so ill that he cannot perform any job. Own job insurance is, or course, more costly.[13]
- Employers may purchase disability insurance for employees. Rates are lower than individual policies due to
  1. No adverse selection
  2. Economies of scale
  3. No risk of non-payment of premium
- Some policies have *return to work assistance*. This may pay for retraining or other items that help the disabled person return to work.

**Long term care insurance** This insurance pays if someone is ill enough to require a home health aid or a nursing home. Generally a 90-day waiting period. Benefits are triggered when one cannot perform 2 or 3 (depending on the policy) **Activities of Daily Living** (ADLs).

There are 6 ADLs:

1. Bathing

[12]But some companies have experimented with offering lower prices for annuities to smokers.
[13]Some disability insurance may be "own job" for a period like 5 years, and "any job" after that.

2. Dressing
3. Eating
4. Toileting (ability to go to/from toilet)
5. Continence (bladder/bowel control)
6. Transferring (ability to go from chair to bed to chair; distinguished from toileting)

Severe cognitive impairment may also trigger benefits.

Some other features are:

- Just like disability, there is a benefit period which may be 2–5 years, or, unlike disability, may be for life.
- Just like disability, there is an off period.
- Benefit payments may be on a reimbursement approach subject to a maximum, or may be a fixed amount per month.

Hybrid life insurance/LTC policies may use one of two approaches:

**Return of premium approach** The excess of premiums over benefit payments is added to the death benefit.[14]

**Accelerated benefit approach** The total LTC benefit cannot exceed the sum insured of the life insurance. The LTC benefits paid are subtracted from the death benefit.

The discussion so far relates to U.S. and Canada. In other countries:

**France** LTC is popular and cheaper because

1. Trigger for most policies is "severe dependency", meaning bed- or chair-bound, stricter than "2 ADL" condition.
2. Policies mostly bought as group insurance.
3. Policies bought at younger ages.
4. Lower average benefits.

Payment is fixed annuity.

**Germany** LTC provided by government social insurance. Additional benefits may be obtained through private LTC insurance. May opt out of government plan (and not pay taxes for it) and use private insurance.

Payment is fixed annuity.

**Japan** Available stand-alone or with whole life insurance. Benefits may increase with increased dependency.

**UK** LTC not offered. Instead, there is an *immediate needs annuity* that one purchases with a single premium when moving to a nursing home. Benefits are a fixed annuity paid directly to nursing home. Insurance company may assume higher-than-standard mortality.

**Critical illness/chronic illness insurance** **Critical illness insurance** pays a lump sum benefit upon a severe condition, such as cancer or heart disease, and then expires.

**Chronic illness insurance** pays a benefit upon an illness from which one may not recover. Benefit may be lump sum or annuity.

Either of these two may be an **accelerated death benefit rider** to a life insurance contract.

[14] I think that for most such policies, there is no other death benefit; the death benefit is just the return of premiums minus benefits.

**Forms of ownership of insurance companies**

1. **Mutual**. Has no stockholders. Profits are distributed to for-profit (participating) policyholders.
2. **Proprietary**[15]. Profits are distributed to shareholders. May have for-profit (participating) policies, and then those policyholders would also get share of profit.

**Continuing care retirement communities** **Continuing care retirement communities** (CCRCs) are communities for seniors who need varying amounts of assistance. They have three levels of support:

1. Independent living units (ILUs) for residents who do not need extensive help. They may offer housekeeping, transportation to shopping, and similar services.
2. Assisted living units (ALUs), that provide more extensive non-medical help, such as cooking and laundering.
3. A skilled nursing facility (SNF) for those needing medical care. This looks more like a hospital.

Some facilities may also have memory care units (MCU) for residents with dementia.

Funding is done through

1. *Full life care*, with a large entry fee and guaranteed monthly payments only increasing with cost of living adjustments. All costs are covered.
2. *Modified life care*, with a lower entry fee and with monthly payments. Monthly fees increase if the resident moves to the ALU, MCU, or SNF, but these increases are less than the full cost.
3. *Fee for service*. Here payments are made for all services provided at the market rates. Entry fees and monthly payments (at least initially) are lowest.

For full and modified life care contracts:

1. They are insurance contracts in the sense that the CCRC assumes the risk of the costs of higher level care. Residents entering with these contracts must be underwritten and healthy enough to qualify. Residents must be able to initially live in the ILU.
2. Some contracts provide a partial refund upon exit.
3. Some CCRCs offer (partial) ownership of the ILU. The resident purchases the unit and upon death or exit it is sold and the resident gets some of the proceeds.

Average age of entry to CCRC in the U.S. is 80. On the average, full life care residents enter at younger ages than modified life care residents, who enter at younger ages than fee-for-service residents.

**Structured settlements** When a person is injured, the responsible party must compensate for the loss. Compensation includes paying for medical treatment and lost wages. Rather than paying a lump sum, the insurance company or the court often provides a **structured settlement**. This settlement may include a lump sum for medical payments and other immediate expenses and a life annuity for lost wages, ongoing medical expenses, and other expenses. Providing a structured settlement is better than paying a lump sum because the payment pattern of the structured settlement better matches the pattern of the losses— the lost wages and the ongoing medical expenses resemble an annuity. Thus it relieves the injured party of two risks:

1. The investment risk—the risk of not getting an adequate return on a lump sum
2. The dissipation risk—the risk that the lump sum will be spent too fast

[15]Called “stock” in the U.S.

A serious injury may require a whole life annuity, whereas a more minor injury would require a temporary life annuity extending until expected recovery.

Structured settlements are commonly used in Workers Compensation insurance (called workers comp for short), an insurance purchased by an employer to pay benefits to workers injured on the job. They are also commonly used in medical malpractice and motor vehicle accidents.

Usually a structured settlement once determined is final. But after severe injury, there may be a period in which payments are made until the time of maximum mortality improvement, at which point a final settlement is made.

**Pensions** A pension is a lump sum or a life annuity paid to a retiree by the employer.

The amount of a pension usually varies with the employee's salary and number of years of service.

Pension plans may be defined contribution or defined benefit. In a **defined contribution plan**, the amount set aside each year of employment is specified.

In a **defined benefit plan**, the annual amount paid to the retiree is specified. Typically the annual amount is $n\alpha S$, where $n$ is the number of years of service, $\alpha$ is an accrual rate specified in the plan, and $S$ is the salary base. $\alpha$ is typically 1–2%. The salary base may be

**Final salary** **The average of salaries in the $k$ years before retirement**.

**Career average salary** **The average of all salaries earned while in service.**

**Career average revalued salaries** The average of all salaries with the salaries indexed for inflation.

Defined benefit plans may have *withdrawal benefits*. These are benefits to terminated employees. Typically these would be computed using the $n\alpha S$ formula, and deferred to retirement age.

Defined contribution plans pay a lump sum of the accumulated funds at retirement. They may be used to purchase an annuity.

**Old SOA Exam LTAM questions**: F18:B4(a), F19:B4(a), S21-1:1,B3(a), F21-A:1, F21-B:1

**Multiple choice sample questions:** 1.1, 1.2

# Part VII

# Practice Exams

Here are 12 practice exams to help you test your knowledge, and to pinpoint areas you are weak in so you will know what to review.

The number of questions on each of the five major topics of Exam FAM-L are in line with the syllabus. The questions are randomly arranged, both in terms of topic and in terms of difficulty. On real exams (or at least the ones they've released), there are some easy questions at the beginning and an easy question at the end, but these exams don't follow that rule.

# Practice Exam 1

**1.** A life age 60 is subject to Gompertz's law with $B = 0.001$ and $c = 1.05$.

Calculate $e_{60:\overline{2}|}$ for this life.

(A) 1.923 (B) 1.928 (C) 1.933 (D) 1.938 (E) 1.943

**2.** Your company sells whole life insurance policies. At a meeting with the Enterprise Risk Management Committee, it was agreed that you would limit the face amount of the policies sold so that the probability that the present value of the benefit at issue is greater than 1,000,000 is never more than 0.05.

You are given:

(i) The insurance policies pay a benefit equal to the face amount $b$ at the moment of death.
(ii) The force of mortality is $\mu_x = 0.001(1.05^x)$, $x > 0$
(iii) $\delta = 0.06$

Determine the largest face amount $b$ for a policy sold to a purchaser who is age 45.

(A) 1,350,000 (B) 1,400,000 (C) 1,450,000 (D) 1,500,000 (E) 1,550,000

**3.** For an annual premium 2-year term insurance on (60) with benefit $b$ payable at the end of the year of death, you are given

(i)

| $t$ | $p_{60+t-1}$ |
|---|---|
| 1 | 0.98 |
| 2 | 0.96 |

(ii) The annual net premium is 25.41.
(iii) $i = 0.05$.

Determine the revised annual net premium if an interest rate of $i = 0.04$ is used.

(A) 25.59 (B) 25.65 (C) 25.70 (D) 25.75 (E) 25.81

**4.** For a fully discrete 25-year term life insurance on (45) with face amount 200,000, you are given:

(i) Mortality follows the Standard Ultimate Life Table.
(ii) Deaths are uniformly distributed between integer years.
(iii) Gross premium payable quarterly is 130.
(iv) Per premium and per policy expenses are

| | Percent of Premium | Per Policy |
|---|---|---|
| First year | 60% | 250 |
| Renewal | 5% | 30 |

(v) Per premium expenses are payable when premiums are payable.
(vi) Per policy expenses are payable at the beginning of each year.
(vii) Cost of settling a death claim is 100.
(viii) $i = 0.05$.

Calculate the gross premium reserve at time 15.

(A) 4817 (B) 4822 (C) 4826 (D) 4892 (E) 4896

**5.** Endowment insurance is no longer offered by major insurers in North America or the UK. Consider the following reasons:

I. The product has low returns.
II. The product is not flexible.
III. There are onerous tax provisions on the product.

State which of these reasons is given in *Actuarial Mathematics for Life Contingent Risks*.

(A) None (B) I and II only (C) I and III only (D) II and III only
(E) The correct answer is not given by (A) , (B) , (C) , or (D) .

**6.** A study is performed on number of days required to underwrite a policy. The results of the study are:

| Number of Days | Number of Policies |
|---|---|
| (0,10] | 11 |
| (10,20] | $x$ |
| (20,50] | $y$ |

An ogive is used to interpolate between interval boundaries.

You are given:

(i) $\hat{F}(15) = 0.35$
(ii) $\hat{f}(15) = 1/30$

Determine $x$.

(A) 16 (B) 18 (C) 20 (D) 22 (E) 24



*Exam questions continue on the next page . . .*

**7.** You are given:

(i) $Z_1$ is the present value random variable for a 10-year term insurance paying 1 at the moment of death of (45).
(ii) $Z_2$ is the present value random variable for a 20-year deferred whole life insurance paying 1 at the moment of death of (45).
(iii) $\mu = 0.02$
(iv) $\delta = 0.04$

Calculate $\mathrm{Cov}(Z_1, Z_2)$.

(A) −0.042 (B) −0.028 (C) −0.023 (D) −0.015 (E) −0.009

**8.** You are given:

(i) For a cohort of 100 newly born children, the force of mortality is constant and equal to 0.01.
(ii) Birthday cards are sent each year to all lives in the cohort beginning on their 80th birthdays, for as long as they live.

Determine the expected number of birthday cards each member of this cohort receives.

(A) 44.7 (B) 44.9 (C) 45.2 (D) 45.5 (E) 45.7

**9.** A special 9-year term insurance on $(x)$ pays the following benefit at the end of the year of death:

| Year of death $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Benefit $b_t$ | 1 | 2 | 3 | 4 | 5 | 4 | 3 | 2 | 1 |

$(DA)^1_{x:\overline{n}|}$ denotes the expected present value of a decreasing term insurance that pays a benefit of $n+1-k$ at the end of the year if death occurs in year $k$, $1 \le k \le n$.

You are given the following expected present values for increasing and decreasing term insurances:

| $n$ | $(IA)^1_{x:\overline{n}\|}$ | $(DA)^1_{x:\overline{n}\|}$ |
|---|---|---|
| 4 | 0.593 | 0.628 |
| 5 | 0.848 | 0.923 |
| 9 | 1.970 | 2.513 |
| 10 | 2.219 | 2.986 |

Determine the expected present value of the special term insurance.

(A) 0.7 (B) 0.8 (C) 1.3 (D) 1.4 (E) 1.8

**10.** For a fully continuous whole life insurance of 1000 on $(x)$:

(i) The gross premium is paid at an annual rate of 25.
(ii) The variance of future loss is 500,000.
(iii) $\delta = 0.06$

Employees are able to obtain this insurance for a 20% discount.

Determine the variance of future loss for insurance sold to employees.

(A) 320,383 (B) 301,261 (C) 442,907 (D) 444,444 (E) 456,253



Exam questions continue on the next page . . .

**11.** You are given that $A_x = 0.4 + 0.01x$ for $x < 60$.

A fully discrete whole life insurance on (30) pays a benefit of 1 at the end of the year of death.

Calculate the net premium reserve at time 20 for this insurance.

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

**12.** You are given the following statements regarding disability insurance.

I. "Own job" insurance tends to be cheaper than "any job" insurance.

II. For a policy with benefit period to 65, longer off periods make the insurance more expensive.

III. The cost of a policy increases as the benefit period increases.

(A) None of I, II, or III is true
(B) I and II only
(C) I and III only
(D) II and III only
(E) The answer is not given by (A), (B), (C), or (D)

**13.** In a mortality study, the cumulative hazard function is estimated using the Nelson-Åalen estimator. There are initially 41 lives. There are no censored observations before the first time of deaths, $t_{(1)}$.

The number of deaths at time $t_{(1)}$ is less than 6.

$\widehat{\mathrm{Var}}\big(\hat{H}(t_{(1)})\big) = 0.000580$.

Determine the number of deaths at time $t_{(1)}$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

**14.** For a fully discrete 20-year deferred whole life insurance of 1000 on (50), you are given:

(i) Premiums are payable for 20 years.
(ii) The net premium is 12.
(iii) Deaths are uniformly distributed between integral ages.
(iv) $i = 0.1$
(v) ${}_9V = 240$ and ${}_{9.5}V = 266.70$.

Calculate ${}_{10}V$, the net premium reserve at the end of year 10.

(A) 272.75 (B) 280.00 (C) 281.40 (D) 282.28 (E) 282.86



Exam questions continue on the next page . . .

**15.** A mortality study begins with 2 lives. You are given the following excerpt of the study data:

| $j$ | $t_{(j)}$ | Deaths at $t_{(j)}$ | Exits in $(t^+_{(j)}, t^-_{(j+1)})$ (censored) | Entrants in $(t^+_{(j)}, t^-_{(j+1)})$ (truncated) |
|---|---|---|---|---|
| 0 | 0 | | 0 | 1 |
| 1 | 3.1 | 1 | 1 | 2 |
| 2 | 4.0 | 1 | 1 | 5 |
| 3 | 5.2 | 1 | 2 | 0 |
| 4 | 6.2 | 1 | 0 | 0 |
| 5 | 8.4 | 1 | 0 | 0 |

Calculate the Nelson-Åalen estimate of $S(7)$.

(A) 0.23 (B) 0.25 (C) 0.27 (D) 0.29 (E) 0.31

**16.** A life age 90 is subject to mortality following Makeham's law with $A = 0.0005$, $B = 0.0008$, and $c = 1.07$.
Curtate life expectancy for this life is 6.647 years.
Using Woolhouse's formula with three terms, compute complete life expectancy for this life.

(A) 7.118 (B) 7.133 (C) 7.147 (D) 7.161 (E) 7.176

**17.** You are given that $\mu_x = 0.002x + 0.005$.
Calculate ${}_{5|}q_{20}$.

(A) 0.015 (B) 0.026 (C) 0.034 (D) 0.042 (E) 0.050

**18.** For a temporary life annuity-due of 1 per year on (30), you are given:

(i) The annuity makes 20 certain payments.
(ii) The annuity will not make more than 40 payments.
(iii) Mortality follows the Standard Ultimate Life Table.
(iv) $i = 0.05$

Determine the expected present value of the annuity.

(A) 17.79 (B) 17.83 (C) 17.87 (D) 17.91 (E) 17.95

**19.** For a mortality table, you are given

(i) Uniform distribution of deaths is assumed between integral ages.
(ii) $\mu_{30.25} = 1$
(iii) $\mu_{30.5} = \frac{4}{3}$

Determine $\mu_{30.75}$.

(A) $\frac{5}{3}$ (B) 2 (C) $\frac{7}{3}$ (D) $\frac{5}{2}$ (E) 3



Exam questions continue on the next page . . .

**20.** In a mortality study on 5 lives, you are given the following information:

| Entry age | Exit age | Cause of exit |
|---|---|---|
| 62.3 | 65.1 | End of study |
| 63.5 | 66.0 | Withdrawal |
| 64.0 | 65.7 | Withdrawal |
| 64.2 | 65.5 | Death |
| 64.7 | 67.7 | End of study |

Assume constant force of mortality between integer ages.

Calculate the maximum likelihood estimate of $q_{65}$.

(A) 0.261 (B) 0.262 (C) 0.263 (D) 0.264 (E) 0.265

*Solutions to the above questions begin on page 923.*

# Practice Exam 1

1. [**Section 6.2**] By formula (5.2),

$$p_{60} = \exp\left(-0.001(1.05^{60})\left(\frac{0.05}{\ln 1.05}\right)\right) = 0.981040$$
$${}_2p_{60} = \exp\left(-0.001(1.05^{60})\left(\frac{1.05^2-1}{\ln 1.05}\right)\right) = 0.961518$$

Then $e_{60:\overline{2}|} = 0.981040 + 0.961518 = \boxed{\mathbf{1.9426}}$. **(E)**

2. [**Lesson 17**] The present value of the benefit decreases with increasing survival time, so the 95th percentile of the present value of the insurance corresponds to the 5th percentile of survival time. The survival probability is

$${}_tp_{45} = \exp\left(-\int_0^t 0.001(1.05^{45+u})\mathrm{d}u\right)$$
$$-\ln {}_tp_{45} = \left.\frac{0.001(1.05^{45+u})}{\ln 1.05}\right|_0^t$$
$$= \frac{0.001(1.05^{45+t} - 1.05^{45})}{\ln 1.05}$$

Setting ${}_tp_{45} = 0.95$,

$$\frac{0.001(1.05^{45+t} - 1.05^{45})}{\ln 1.05} = -\ln 0.95$$
$$1.05^{45+t} = (-1000 \ln 0.95)(\ln 1.05) + 1.05^{45} = 11.48762$$
$$1.05^t = \frac{11.48762}{1.05^{45}} = 1.27853$$
$$t = \frac{\ln 1.27853}{\ln 1.05} = 5.0361$$

The value of $Z$ if death occurs at $t = 5.0361$ is $be^{-5.0361(0.06)}$, so the largest face amount is $1{,}000{,}000e^{5.0361(0.06)} = \boxed{\mathbf{1{,}352{,}786}}$. **(A)**

3. [**Lesson 26**] The revised premium for the entire policy is 25.41 times the ratio of the revised premium per unit at 4% to the original premium per unit at 5%.

We calculate the original net premium per unit, $P^{1}_{60:\overline{2}|}$.

$$\ddot{a}_{60:\overline{2}|} = 1 + \frac{0.98}{1.05} = 1.93333$$
$$A^{1}_{60:\overline{2}|} = \frac{0.02}{1.05} + \frac{(0.98)(0.04)}{1.05^2} = 0.054603$$
$$P^{1}_{60:\overline{2}|} = \frac{A^{1}_{60:\overline{2}|}}{\ddot{a}_{60:\overline{2}|}} = \frac{0.054603}{1.93333} = 0.028243$$

Now we recalculate at 4%. Call the revised premium $P'^{1}_{60:\overline{2}|}$.

$$\ddot{a}_{60:\overline{2}|} = 1 + \frac{0.98}{1.04} = 1.94231$$
$$A^{1}_{60:\overline{2}|} = \frac{0.02}{1.04} + \frac{(0.98)(0.04)}{1.04^2} = 0.055473$$
$$P'^{1}_{60:\overline{2}|} = \frac{0.055473}{1.94231} = 0.028561$$

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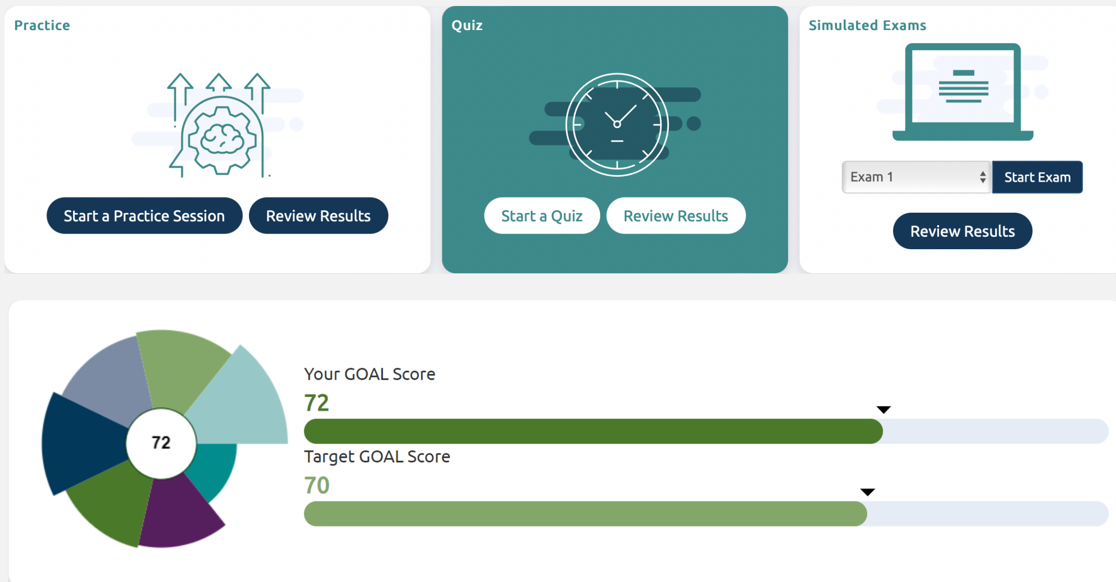


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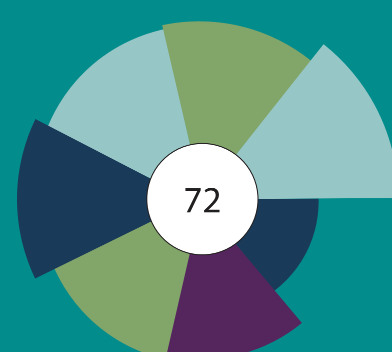


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